

<sup>1</sup>*Professor Barbara Namysłowska-Wilczyńska,  
Institute of Geotechnics and Hydrotechnics,  
University of Technology, Wrocław, Poland*

<sup>2</sup>*Bożena Skorupska Ph.D., E. Eng., Institute of Non-Ferrous Metals, Gliwice, Poland*

<sup>3</sup>*Andrzej Wieniewski Ph.D., E. Eng., Institute of Non-Ferrous Metals, Gliwice, Poland*

## **GEOSTATISTICAL ANALYSIS OF VARIATION IN TECHNOLOGICAL PARAMETERS OF ASH-SLAGS**

### **Introduction**

The aim of the spatial analysis was to develop a 3D geostatistical model of the variation in the parameters characterizing a power industry ash slags heap (comprising reservoirs A and B) in Świecie (near Toruń) in the Kujawy-Pomerania Province (Poland). The various applications of geostatistical (estimation and simulation) methods in many disciplines and research fields, often to solve interdisciplinary problems, are widely described in the literature [ 1, 2, 5, 12]. One should mention here earth sciences (mainly geological sciences and mining) [5, 6, 8, 9, 10, 12], environmental protection [3, 4, 5], climatology [7, 12], geophysics, agricultural sciences [12], geochemistry, oceanology, epidemiology, meteorology, forestry, materials science and even power engineering [5, 11]. The wide capabilities of the tools of geostatistics, enabling one to model regionalized variables, estimate averages  $Z^*$  and predict simulated values  $Z_S$  of the analyzed parameters characterizing various phenomena, as well as the universality of the methodological approach encourage researchers to try out these techniques in new research areas. A major advantage of the geostatistical methods is that when estimating averages  $Z^*$  and  $Z_S$  one can obtain minimum estimation variance  $\sigma_k^2$  or simulation variance  $\sigma^2$ , and many other parameters describing the effectiveness and quality of the estimations.

Geostatistical studies of four technological parameters of the ash slags were carried out by the author<sup>1</sup> as part of her collaboration with the Institute of Nonferrous Metals in Gliwice within the framework of the IT I (KB/140/12504/IT1-B/U/08) project entitled “Development of technology of waste dump ash slag utilization through carbon separation by froth flotation” run by the Mineral Raw Materials Processing and Wastes Utilization Department of INM [13, 14].

Because of the paper's space confines only selected results of the 3D analyses of the variation in the ash slags heap's technological parameters (for reservoir A) are presented. These are the results of the modelling of moisture content  $W$  and ash content  $A_O$ , using the directional variogram function and estimating averages  $Z^*$  by means of ordinary (block) kriging. Reference is made to the main statistics concerning organic carbon content  $C_{org.}$  and beneficiation capacity  $e'$  of the ash slags as well as to the investigation of the interdependence between the values of the technological parameters<sup>1</sup> for reservoir A [9-10].

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<sup>1</sup> Namysłowska-Wilczyńska B., “Geostatistical analysis of the variation in the technological parameters of ash slags for a selected waste heap”. Report of the Institute of Geotechnics and Hydrotechnics at Wrocław University of Technology [in Polish], series SPR, No. 8, Wrocław 2008, p. 115.

## Range of studies

The statistical and geostatistical studies<sup>2</sup> covered 150 m x 400 m x 10 m reservoir A within the ash-slag heap area (fig. 1). The specific gravity of the material in this reservoir was 1.948 g/cm<sup>3</sup>. Samples for the studies were taken from twelve 1-10 m deep exploratory boreholes made within the reservoir area boundaries (fig. 1). The statistical sample size was n = 120.

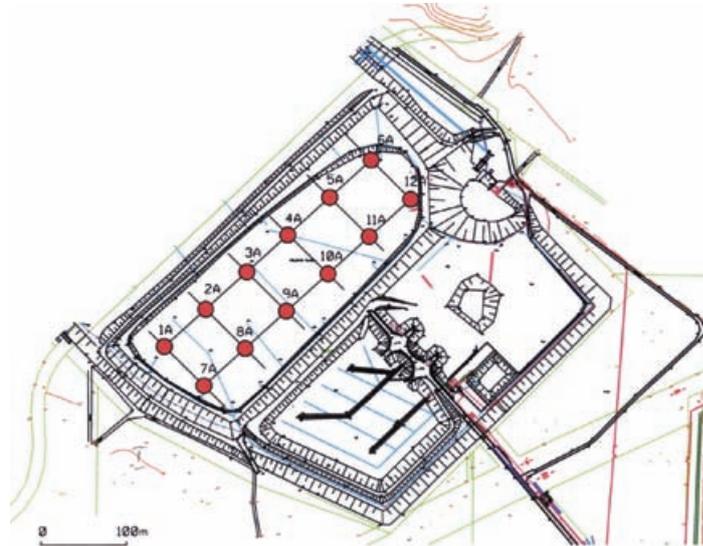


Fig. 1. Map showing locations of 12 boreholes within boundaries of ash slags heap (reservoir A) near Świecie (Toruń).

The ash slags' ash content  $A_0$ , moisture content  $W$  and beneficiation capacity  $e'$  were determined by the Laboratory of the Mineral Raw Materials Processing and Wastes Utilization Department of INM in Gliwice while its organic carbon content  $C_{org.}$  was determined by the Analytical Chemistry Laboratory of INM.

Beneficiation capacity  $e'$  is defined as the amount of carbon recovered from 1 t of ash slags. Parameter  $e'$  is calculated as the product of the carbon yield and the carbon content in a given sample.

An input database (in 3D) concerning the technological parameters was developed. It contains the values of coordinates X, Y, Z (depth) specifying the sample taking locations, and the values of the four investigated regionalized variables. The content of this database was the basis for spatial analyses.

The object of the analyses was the variation in the values of two parameters characterizing the ash slags heap, i.e. moisture content  $W$  and ash content  $A_0$ . The main statistics and distribution histograms concerning the parameters were estimated and the spatial correlation between their values was studied.

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<sup>2</sup> The statistical and geostatistical analyses were carried out using selected computer programs included in the ISATIS software package version 2011.2 (Geovariances & Ecole Des Mines de Paris in Avon Cedex, France) (Isatis 2001).

## Estimation of main statistics of technological parameters

The technological parameters of the ash slags vary markedly within the boundaries of the heap (reservoir A). The evaluations of the main statistics, based on original data, indicate their different behaviour depending on the kind of parameter (tab. 1). Ash content  $A_o$  and moisture content  $W$  show respectively small (variation coefficient  $V=8\%$ ) and average variation ( $V=25\%$ ). Whereas organic carbon content  $C_{org}$  and beneficiation capacity  $e'$  were found to vary greatly ( $V=53-59\%$ ).

**Table 1** Main statistics of technological parameters of ash slags in waste heap.

Analyzed parameter	Sample size n	Minimum value $X_{min}$ [%]	Maximum value $X_{max}$ [%]	Average value $X$ [%]	Standard deviation $S$ [%]	Variation coefficient $V$ [%]
Ash content $A_o$	120	65.14	95.78	84.02	6.49	8
Moisture content $W$	120	8.84	36.43	23.64	6.01	25

The histogram of ash content  $A_o$  shows left-side asymmetry. A similar (but weaker) distribution trend with noticeable negative skewness is visible in the moisture content  $W$  histogram. Skewness coefficients  $g_1$  of the histograms are in a range of 0.28 ( $W$ ) ÷ - 0.53 ( $A_o$ ). The histograms of organic carbon content  $C_{org}$  and beneficiation capacity  $e'$  show right-side asymmetry, with positive skewness coefficients  $g_1$  ranging from 0.88 ( $C_{org}$ ) to 0.94 ( $e'$ ). Flattening (kurtosis) coefficients  $g_2$  of the histograms amount to 2.40 - 3.49.

## Study of correlation between technological parameters

Linear correlation coefficients  $r$  indicate spatial correlation between the values of the investigated parameters (tab. 2). The spatial correlation varies within the area where the ash slags occur. There is strong inverse (negative) correlation between respectively ash content  $A_o$  and organic carbon content  $C_{org}$  ( $r = - 0.75$ ), ash content  $A_o$  and moisture content  $W$  ( $r = - 0.75$ ), and ash content  $A_o$  and beneficiation capacity  $e'$  ( $r = - 0.70$ ). The strongest positive correlation ( $r = 0.98$ ) was found between carbon content  $C_{org}$  and beneficiation capacity  $e'$ .

**Table 2** Linear correlation coefficients  $r$  of technological parameters of waste heap ash slags

Analyzed parameter	Sample size n	Ash content $A_O$ [%]	Organic carbon content $C_{org.}$ [%]	Moisture content $W$ [%]	Beneficiation capacity $e'$ [kg/t <sub>mat.</sub> ]
Ash content $A_O$ [%]	120	1.00	-0.75	-0.75	-0.70
Organic carbon content $C_{org.}$ [%]	120	-0.75	1.00	0.67	0.98
Moisture content $W$ [%]	120	-0.75	0.67	1.00	0.67
Beneficiation capacity $e'$ [kg/t <sub>mat.</sub> ]	120	-0.70	0.98	0.67	1.00

### Structural analysis of variation in technological parameters

The structure of the spatial variation in the technological parameters was studied through a structural (variographic) analysis using variogram function  $\gamma(h)$ . A clear rising trend of function  $\gamma(h)$  is observed in the directional variograms calculated depending on the sample taking depth. The strongest rising trend of  $\gamma(h)$  is observed in the variograms of organic carbon content  $C_{org.}$  and beneficiation capacity  $e'$  [9-10]. The variograms were approximated with a spherical model ( $e'$ ) or a spherical model and a linear model ( $C_{org.}$ ) or with the two models combined with nugget effect  $C_O$  (ash content  $A_O$  and moisture content  $W$ ) (tabs 3-4). The correlation distance, i.e. the variogram range ( $a = 76.89$  m,  $a = 95.54$  m), was found to be much longer for  $C_{org.}$  than for beneficiation capacity  $e'$  ( $a = 13.22$  m), moisture content  $W$  ( $a = 14.42$  m) and ash content  $A_O$  ( $a = 13.47$  m,  $a = 11.26$  m) (tabs 3-4). Nugget effect  $C_O$  was found in the variograms of ash content  $A_O$  and moisture content  $W$  (tabs 3-4), being more pronounced in the  $A_O$  variogram. No  $C_O$  effect occurred in the variograms of  $C_{org.}$  and  $e'$ , which indicates the absence of abrupt changes in function  $\gamma(h)$ .

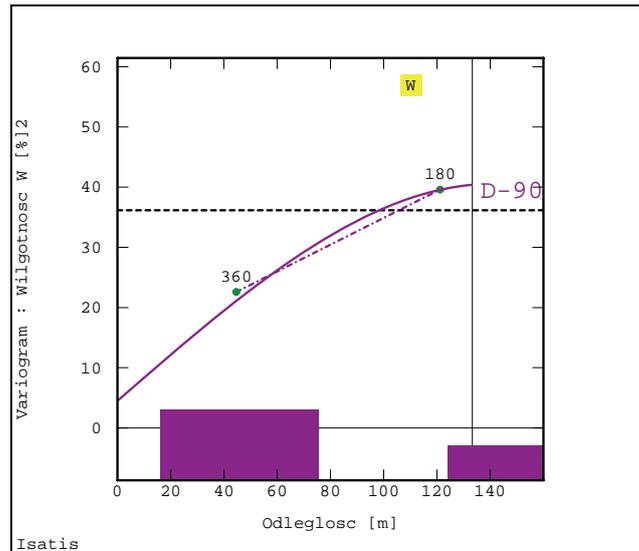


Fig. 2. Directional variogram of moisture content  $W$  [%]<sup>2</sup> in heap ash slags, with fitted theoretical model

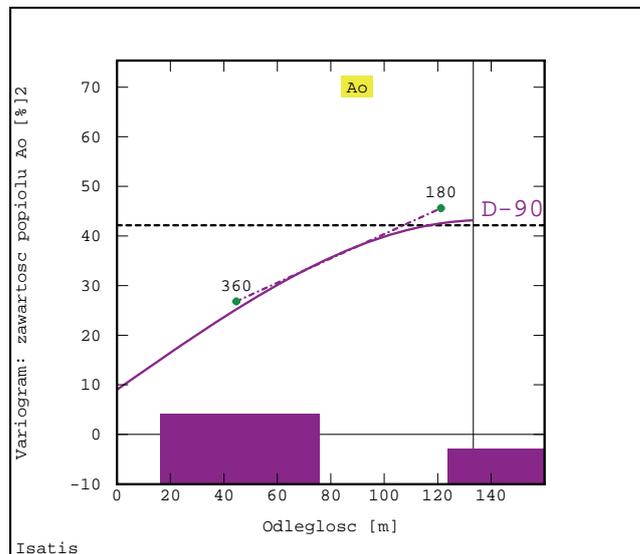


Fig. 3. Directional empirical variogram of ash content  $A_o$  [%]<sup>2</sup> in heap ash slags, with fitted theoretical model.

**Table 3** Geostatistical parameters of models of directional (along axis Z) variograms of heap ash slags technological parameters.

Analyzed parameters	Nugget effect $C_0$ [%] <sup>2</sup>	Variance component $C'$ [%] <sup>2</sup>	Sill variance $C$ [%] <sup>2</sup>	Range of influence $a$ [m]	Main model structures
Moisture content $W$	4.5	34.20 3.61	42.31	14.42 14.42	spherical, linear

**Table 4** Geostatistical parameters of models of directional (along axis Z) variograms of heap ash slags technological parameters.

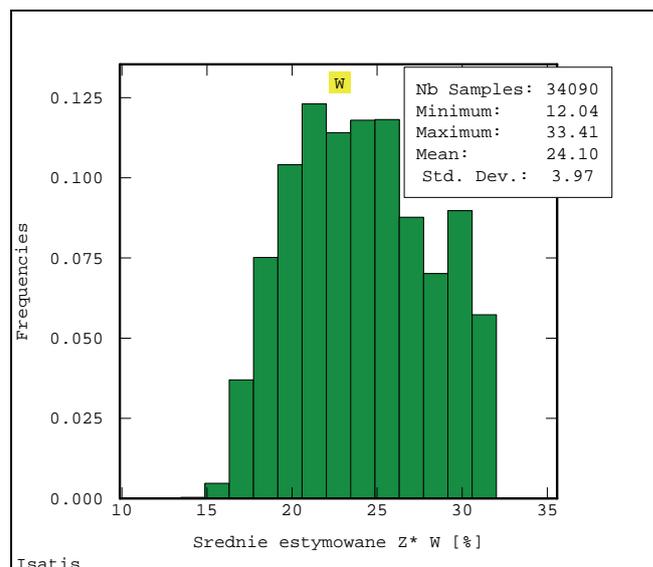
Analyzed parameter	Nugget effect $C_0$ [%] <sup>2</sup>	Variance component $C'$ [%] <sup>2</sup>	Sill variance $C$ [%] <sup>2</sup>	Range of influence $a$ [m]	Main model structures
Ash content $A_0$	9.01	28.13 4.22 4.22	36.57	13.47 11.26 11.26	spherical, spherical, linear

### Estimation of technological parameters

In the next stage of the geostatistical studies ordinary (block) kriging was applied. Averages  $Z^*$  and estimation standard deviation  $\sigma_k$  were estimated (in 3D) for the particular ash slags deposition depths (1-10 m) within the boundaries of the reservoir. A (3D) spatial grid with a 5 m x 5 m x 1 m elementary block was adopted for the kriging calculations. The total number of grid nodes covered by the estimation amounted to 34090 (63, 59 and 10 along respectively axis X, Y and Z). The estimation slicing height along axis Z was 1 m.

The values of the geostatistical parameters of the theoretical models representing the directional variograms of moisture content W and ash content  $A_0$  (tables 3-4) were taken into account. New databases of (input) grid data, containing 34090 values for respectively: elementary grid block centre coordinates X, Y and Z, estimated averages  $Z^*$  and estimation standard deviations  $\sigma_k$ , were obtained. The grid data can be processed and serve as the basis for further spatial analyses and the visualization of their results, e.g. in the form of raster maps and isoline and gradient maps, grid cross sections, etc.

### Results of estimating moisture content W averages $Z^*$ in spatial grid nodes



**Fig. 4.** Histogram of estimated averages  $Z^*$  of moisture content W [%] in heap ash slags.

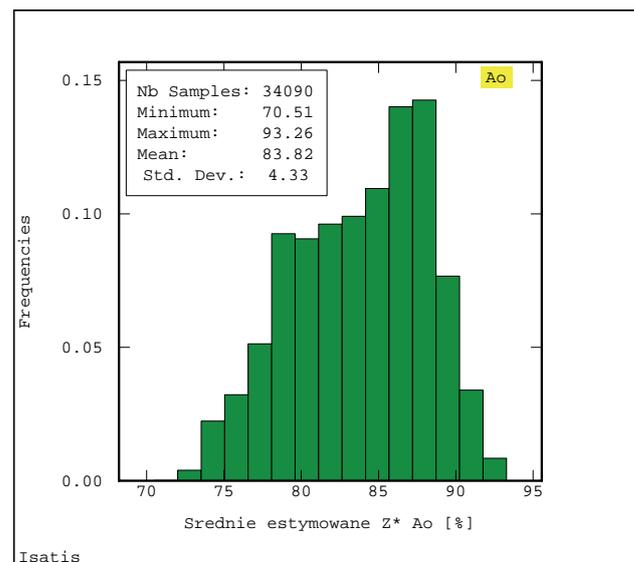
The histogram of moisture content W averages  $Z^*$  is close to symmetrical (fig. 4.) with coefficient  $g_1$  amounting to merely 0.08. The moisture content histogram calculated using the original data shows a slight tendency towards negative skewness.

Variation coefficients  $V$  of estimated averages  $Z^*$ , and estimation standard deviations  $\sigma_k$  of moisture content  $W$  [%] indicate that the variation of the two parameters is small (tab. 5).

**Table 5** Global statistics of geostatistical parameters of moisture content  $W$  in heap ash slags, in elementary grid nodes (ordinary block kriging).

Geostatistical parameter	Number of grid nodes $n$	Minimum estimated value $X_{\min}$ [kg/t <sub>mat.</sub> ]	Maximum estimated value $X_{\max}$ [kg/t <sub>mat.</sub> ]	Average value $X$ [kg/t <sub>mat.</sub> ]	Standard deviation $S$ [kg/t <sub>mat.</sub> ]	Variation coefficient $V$ [kg/t <sub>mat.</sub> ]
Estimated average $Z^*$	34090	12,04	33.41	24.1	3.97	16
Estimation standard deviation $\sigma_k$	34090	1.92	10.55	7.23	1.15	16

### Results of estimating averages $Z^*$ of ash content $A_0$ in nodes of spatial elementary grid



**Fig. 5.** Histogram of estimated averages  $Z^*$  of ash content  $A_0$  [%] in heap ash slags.

The histogram of averages  $Z^*$  of ash content  $A_0$  shows a marked skewness, with coefficient  $g_1$  amounting to 0.32. A similar tendency towards left-side asymmetry is noticeable in the histogram calculated on the basis of the original ash content data.

Variation coefficient  $V$  of estimated averages  $Z^*$  of ash content  $A_0$  indicates that the variation of this parameter is very small (tab. 6). A higher (low to average) variation coefficient  $V$  was obtained for estimation standard deviation  $\sigma_k$ .

The variation in averages  $Z^*$  of beneficiation capacity  $e'$ , and to a smaller degree in those of carbon content  $C_{\text{org.}}$ , is stronger than the very small variation in averages  $Z^*$  of ash content  $A_0$  and in those of moisture content  $W$ .

**Table 6** Global statistics of geostatistical parameters of ash content  $A_O$  in heap ash slags, in elementary spatial grid nodes (block kriging).

Geostatistical parameter	Number of grid nodes <b>n</b>	Minimum estimated value $X_{\min}$ [%]	Maximum estimated value $X_{\max}$ [%]	Average value <b>X</b> [%]	Standard deviation <b>S</b> [%]	Variation coefficient <b>V</b> [%]
Estimated average $Z^*$	34090	70.51	93.26	83.82	4.33	5
Estimation standard deviation $\sigma_k$	34090	2.26	11.78	7.67	7.41	19

### Description of estimation results and their 3D visualization

Raster maps of estimated averages  $Z^*$  and estimation standard deviations  $\sigma_k$  of ash content  $A_O$  and moisture content  $W$  in the ash slags make it possible to trace in detail the variation in the above parameters within the boundaries of the heap (reservoir A) as well as from its shallowest layers down to its deepest layer. The raster images of the variation also enable one to see if there is a spatial correlation between the values of the considered parameters.

### Moisture content $W$ in ash slags

Raster maps of the distribution of estimated ash slags moisture content  $W$  averages  $Z^*$ , in perspective view, are shown in figs 6-7.

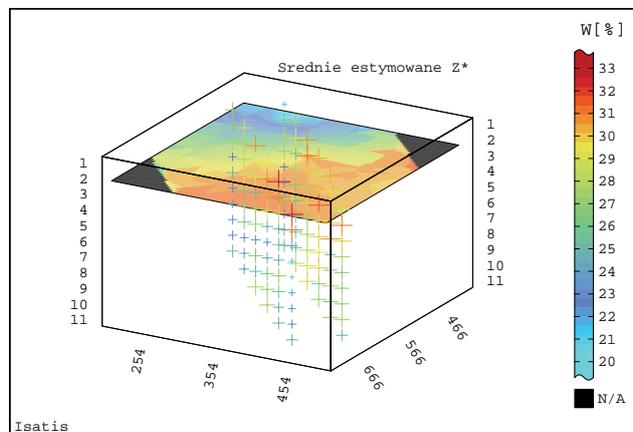


Fig. 6.

Raster map of estimated heap ash slags moisture content  $W$  averages  $Z^*$ , in perspective view (depth 2m).

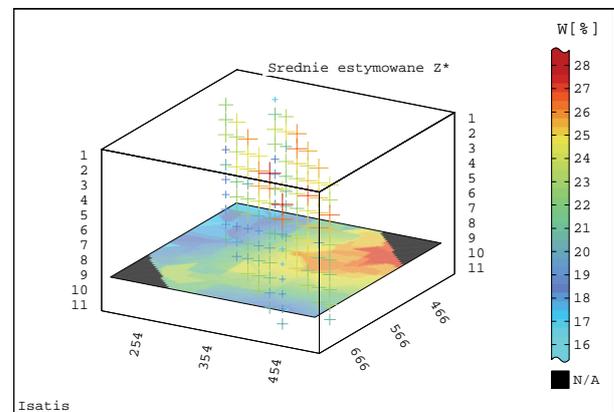


Fig. 7.

Raster map of estimated heap ash slags moisture content  $W$  averages  $Z^*$ , in perspective view (depth 9 m).

Two subareas of variation can be distinguished on the raster map of moisture content  $W$  averages  $Z^*$  calculated for the depth of 2 m (fig. 6). They extend along the diagonal of the considered area. One of the subareas represents lower ( $Z^*$ : 19.86 ÷ 27.29%) and intermediate ( $Z^*$ : 28.03 ÷ 28.78%) moisture content  $W$  averages. The other (larger) subarea is characterized by higher averages  $Z^*$  ( $Z^*$ : 29.52 ÷ 31.75%). Also a small centre with the lowest average  $Z^*$  is visible in the same spatial location.

In the raster map for the depth of 9 m (fig. 7) a large uniform subarea with the highest average  $Z^*$  is clearly distinct. The highest averages  $Z^*$  are in the range  $Z^*$ : 25.04 ÷ 27.16% (fig. 7). Lower averages  $Z^*$  are in intervals: 15.89 ÷ 20.12% and 20.12 ÷ 22.93%.

The spatial location of small centres of elevated carbon content  $C_{org.}$  coincides with the extent of the zones of low ash content  $A_0$  while subareas of lower carbon content  $C_{org.}$  occur in conjunction with subareas characterized by higher ash content  $A_0$ .

**Maximum estimated moisture content W averages  $Z^*$**

<b>1 m</b>	29.94 ÷ 32.87 [%]
<b>2 m</b>	29.52 ÷ 31.75 [%]
<b>3 m</b>	29.53 ÷ 31.52 [%]
<b>4 m</b>	28.50 ÷ 30.45 [%]
<b>5 m</b>	27.85 ÷ 31.20 [%]
<b>6 m</b>	27.54 ÷ 31.20 [%]
<b>7 m</b>	26.13 ÷ 29.38 [%]
<b>8 m</b>	24.81 ÷ 27.34 [%]
<b>9 m</b>	25.04 ÷ 27.16 [%]
<b>10 m</b>	25.25 ÷ 27.30 [%]

**Minimum estimated moisture content W averages  $Z^*$**

17.21 ÷ 20.15 [%]
19.86 ÷ 22.09 [%]
20.94 ÷ 22.92 [%]
20.06 ÷ 22.01 [%]
13.32 ÷ 16.67 [%]
11.69 ÷ 15.35 [%]
12.04 ÷ 15.29 [%]
13.86 ÷ 16.39 [%]
15.89 ÷ 18.00 [%]
16.39 ÷ 18.44 [%]

The lower limits of the ranges of maximum estimated ash slags moisture content W averages  $Z^*$  consistently get lower in the interval of 1-10 m (figs 6-7). The upper limits of averages  $Z^*$  remain at approximately similar level down to the depth of 6 m, and then get markedly lower in the interval of 6-10 m, i.e. in the reservoir's deeper layers. The lower limits of minimum moisture content W averages  $Z^*$  get higher to the depth of 4 m and then get lower, but inconsistently (figs 6-7). From the depth of 8 m minimum averages  $Z^*$  (both the upper and lower limits) increase again.

**Ash content  $A_0$  in ash slags**

Raster maps of the distribution of estimated ash slags ash content  $A_0$  [%] averages  $Z^*$ , in the perspective view, are shown in figs 8-9.

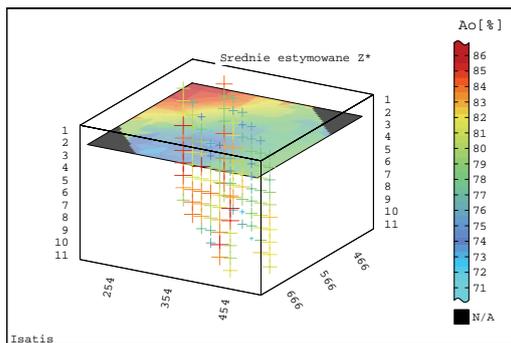


Fig. 8. Raster map of distribution of estimated heap ash slags  $A_0$  content [%] averages  $Z^*$ , in perspective view (depth of 2 m)

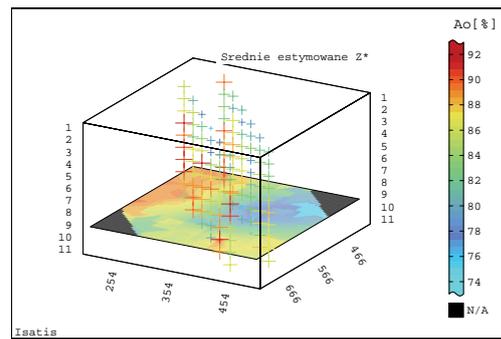


Fig. 9. Raster map of distribution of estimated heap ash slags  $A_0$  content [%] averages  $Z^*$ , in perspective view (depth of 9 m)

Subareas with a much higher ash content occupy a larger surface area on the raster maps. The  $A_0$  content there ranges from 83.11 to 85.89% at the depth of 2 m (fig. 8) and from 88.10 to 91.33% at the depth of 9 m (fig. 9). On the map of averages  $Z^*$  of  $A_0$  content calculated for the depth of 2 m there is a subarea characterized by a lower  $A_0$  content (69.80 ÷ 74.08% and 71.07 ÷ 74.78%), with a single small centre with the lowest  $A_0$  content (fig. 8). The intensity of  $A_0$  content increases with ash slags

heap depth. A distinct centre with higher  $A_O$  content (88.10 ÷ 91.33%) is observed, e.g., at a depth of 9 m (fig. 9).

Centres with lower and higher ash content  $A_O$  correspond to zones with higher organic carbon content  $C_{org.}$  and elevated beneficiation capacity  $e'$ , and vice versa. Higher moisture content  $W$  in the ash slags is found in places characterized by higher carbon content  $C_{org.}$  and higher beneficiation capacity  $e'$ .

	<b>Maximum estimated ash content <math>A_O</math> averages <math>Z^*</math></b>	<b>Minimum estimated ash content <math>A_O</math> averages <math>Z^*</math></b>
<b>1 m</b>	83.71 ÷ 86.92 [%]	69.80 ÷ 73.01 [%]
<b>2 m</b>	83.11 ÷ 85.89 [%]	71.07 ÷ 73.85 [%]
<b>3 m</b>	86.03 ÷ 88.73 [%]	74.32 ÷ 77.02 [%]
<b>4 m</b>	87.72 ÷ 90.49 [%]	75.70 ÷ 78.48 [%]
<b>5 m</b>	89.99 ÷ 93.13 [%]	76.37 ÷ 79.52 [%]
<b>6 m</b>	89.79 ÷ 92.77 [%]	76.86 ÷ 79.84 [%]
<b>7 m</b>	90.47 ÷ 93.41 [%]	77.71 ÷ 80.65 [%]
<b>8 m</b>	89.51 ÷ 91.97 [%]	78.82 ÷ 81.28 [%]
<b>9 m</b>	88.10 ÷ 91.33 [%]	74.10 ÷ 77.33 [%]
<b>10 m</b>	87.75 ÷ 91.09 [%]	73.30 ÷ 76.64 [%]

The ranges of maximum ash content  $A_O$  averages  $Z^*$  clearly increase with depth from 1 to 10 m (figs 8-9). Maximum averages  $Z^*$  reach higher values at deeper levels, being slightly lower at the depth of 8 m and stabilizing at the depth of 10 m. This observation also applies to the ranges of minimum averages  $Z^*$  (figs 8-9). Ash content  $A_O$  reaches markedly lower values at the depth of 9-10 m.

The variation of the three studied parameters, i.e. organic carbon content  $C_{org.}$ , moisture content  $W$  and beneficiation capacity  $e'$ , shows a certain regularity. Maximum averages  $Z^*$  and on the whole minimum averages  $Z^*$ , especially their upper limits, consistently decrease with depth. Whereas the ranges of maximum ash content averages  $Z^*$  consistently increase, reaching higher values of  $Z^*$  in the deeper layers of the ash slags.

The highest averages  $Z^*$  of carbon content  $C_{org.}$  and beneficiation capacity  $e'$  characterize the surface (1-2 m) and near-surface (3-4 m) layers of the investigated ash slags heap. Markedly smaller averages  $Z^*$  were obtained for the deeper layers situated at a depth of 5-8 m. The lowest carbon content  $C_{org.}$  and beneficiation capacity  $e'$  were found to characterize the deepest layers situated at a depth of 9-10 m.

The low and very low variation coefficients  $V$  of averages  $Z^*$  of moisture content  $W$  ( $V=16\%$ ) and ash content  $A_O$  ( $V=5\%$ ) indicate slightly varied behaviour of the parameters. Such estimations of averages  $Z^*$  are accompanied by relatively low coefficients  $V$  of estimation standard deviation  $\sigma_k$ , ranging from 16 to 19%, which is indicative of the reliability of the obtained kriging estimates.

It should be noted that the layers of ash slags in reservoir A, showing the highest beneficiation capacity  $e'$ , are utilizable and so can be of interest to some sectors of economy (mining, environmental protection, the power industry).

### **Conclusion**

A 3D geostatistical model of the variation in the technological parameters of the ash slags in a power industry waste heap was developed. The model was based on the directional variogram function and ordinary (block) kriging. Thanks to the use of the above methods the analyzed parameters were estimated both along the heap's horizontal extent and its depth. On this basis full graphic documentation of the identified variation, in the form of raster maps, isoline maps

and block diagrams of estimated averages  $Z^*$ , estimation standard deviation  $\sigma_k$  and other parameters characterizing the effectiveness of the estimations, was created.

Owing to the use of the geostatistical methods to describe the variation in the technological parameters of the ash slags (considered as an anthropogenic deposit) a collection of data valuable from research, methodological and practical point of view was acquired. The data can be very useful for a project aimed at utilizing the deposited ash slags.

The presented research methodology is universal and can be applied to solve various problems in such fields as geology and mining, the power industry, environmental protection (to estimate the heavy metals, toxic substances and deleterious substances content in soils and underground waters), the spatial characterization of lands to be revitalized, contaminated land remediation and climatology (to forecast the values of climatic and environmental parameters, such as the amount of rainfall and the amount of chemical compounds present in rain, as determined by the natural environment conditions).

## References

- [1] Armstrong M.: *Basic Linear Geostatistics*. Springer, Berlin 1998.
- [2] Isaaks E. H. Srivastava R. M.: *An Introduction to Applied Geostatistics*. OUP, New York 1989.
- [3] Namysłowska-Wilczyńska B., Rusak K.: *Geostatistics description of heavy metal content variation in soil of Olkusz region against a background of geological structure by GIS method*. *Mathematische Geologie, Volume 7. Mathematical Methods Applied to Geology and Mining. Mathematische Methoden in Geologie und Bergbau*. CPress Verlag. Band 7, Dresden 2003.
- [4] Namysłowska-Wilczyńska B., Pyra J.: *Analiza przestrzenna zawartości miedzi w środowisku gruntowo-wodnym Legnicko-Głogowskiego Okręgu Miedziowego*. *Roczniki Geomatyki (Annals of Geomatics), Tom III, Zeszyt 4, Polskie Towarzystwo Informatyki Przestrzennej, Warszawa 2005*.
- [5] Namysłowska-Wilczyńska B.: *Geostatystyka - Teoria i Zastosowania*. Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2006.
- [6] Namysłowska-Wilczyńska B.: *Analizy przestrzenne z zastosowaniem metod geostatystycznych. Modelowanie 3D porfirowego złoża miedzi Rio Blanco w Peru*. *Roczniki Geomatyki (Annals of Geomatics), Polskie Towarzystwo Informatyki Przestrzennej, Tom V, Zeszyt 1, Warszawa 2007*.
- [7] Namysłowska-Wilczyńska B.(ed.): *Modelowanie i prognozowanie (3D) wysokości opadu atmosferycznego i zawartości siarczanów w opadzie dla obszaru dorzecza środkowej Odry z zastosowaniem geostatystyki*. Praca zbiorowa - Modelowanie Procesów Hydrologicznych, Oficyna Wydawnicza Politechniki Wrocławskiej. Wrocław 2008.
- [8] Namysłowska-Wilczyńska B.: *3D geostatistical modelling of parameters of Rio Blanco porphyritic copper deposit in Peru*. *Proceedings - Primer Seminario Internacional de Geologia para la Industria Minera, Editors: Irene Aracena, Carmen Holmgren & Romke Kuyvenhoven, GEOMIN 2009, GECAMIN, 10 – 12 June 2009, Antofagasta, Chile, CAPITULO 05, Geoestadística, 2009, pp. 301 – 325*.
- [9] Namysłowska-Wilczyńska B., Skorupska B., Wieniewski A., *Application of geostatistical methods to describe variation in technological parameters of ash-slags*. *PROCEEDINGS OF LINNAEUS ECO-TECH '10 22 - 24 November, 2010 KALMAR, SWEDEN, Edited by: Fabio Kaczala, Sandrine Arzur, Ida Tjader, William Hogland, pp. 607 – 618; 7 th International Conference on Natural Sciences and Technologies for Waste and Wastewater Treatment, Remediation, Emissions Related to Climate, Environ-*

mental and Economic Effects - ECO-TECH' 10, University w Kalmar; Session Waste to Energy, 23.11.2010. Book of Abstracts, p. 147.

- [10] Namysłowska-Wilczyńska B., Skorupska B., Wieniewski A., *Analiza przestrzenna wzbogacalności popiołów z wykorzystaniem metod geostatystycznych*. *Górnictwo i Geoinżynieria* Rok 35 Zeszyt 2 2011, ss. 451 – 470.
- [11] Namysłowska - Wilczyńska B., Wilczyński A.: *3D electrical power demand forecasting as tool for planning electrical power firm's activity by means of geostatistical methods*. Zeszyt Prac Naukowych Uniwersytetu Ekonomicznego (UE), No. 91, 'Ekonometria 28. Forecasting', Wrocław 2010.
- [12] Wackernagel H.: *Multivariate Geostatistics, An Introduction with Applications*. 2 nd edition, Springer – Verlag Berlin, Heidelberg, New York 1998.
- [13] Wieniewski A, Skorupska B. i inni: *Opracowanie technologii utylizacji popiołów ze składowisk poprzez wydzielenie węgla metodą flotacji pianowej*; Projekt Inicjatywa Technologiczna I, Sprawozdanie IMN 6656/1-12/09, Gliwice 2009.
- [14] Wieniewski A. , Skorupska B., Mazurek D.: *Technologia utylizacji popiołów ze składowisk poprzez wydzielenie węgla metodą flotacji pianowej*. XVII Międzynarodowa Konferencja „Popioły w energetyce”, Zakopane 2009.